Jurong Pioneer Junior College
H2 Mathematics **JC2-2025**
Preliminary Exam Paper 2 Solution

Q1

(i)

$\dfrac{\mathrm{d}x}{\mathrm{d}t} = k\left(P - x\right), k > 0$

$\displaystyle\int \frac{1}{P-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$

$-\ln\left(P-x\right) = kt + C$ since $P > x$

$\ln\left(P-x\right) = -kt - C$

$P - x = \mathrm{e}^{-kt-C}$

$P - x = \mathrm{e}^{-C} \bullet \mathrm{e}^{-kt}$

$P - x = A\mathrm{e}^{-kt},$ $A = \mathrm{e}^{-C}$

$x = P - A\mathrm{e}^{-kt}$

When $t = 0,$ $x = 0 \Rightarrow A = P$

$x = P - P\mathrm{e}^{-kt}$

$x = P\left(1 - \mathrm{e}^{-kt}\right)$ (Shown)

(ii)

When $t = 12,$ $x = \dfrac{1}{2}P$

$\dfrac{1}{2}P = P\left(1 - \mathrm{e}^{-12k}\right)$

$\mathrm{e}^{-12k} = \dfrac{1}{2}$

$-12k = \ln\dfrac{1}{2} = -\ln 2$

$k = \dfrac{1}{12}\ln 2$

$x = P\left(1 - \mathrm{e}^{-\frac{t}{12}\ln 2}\right)$

When $x = 0.8P$,

$0.8P = P\left(1 - \mathrm{e}^{-\frac{t}{12}\ln 2}\right)$

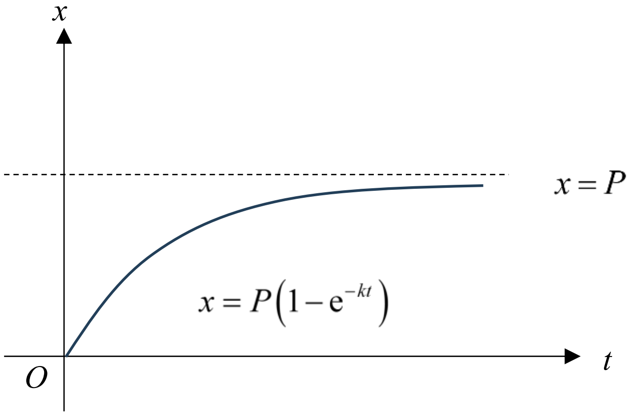
$\mathrm{e}^{-\frac{t}{12}\ln 2} = 0.2$

$-\dfrac{t}{12}\ln 2 = \ln 0.2$

$t = \dfrac{-12\ln 0.2}{\ln 2}$

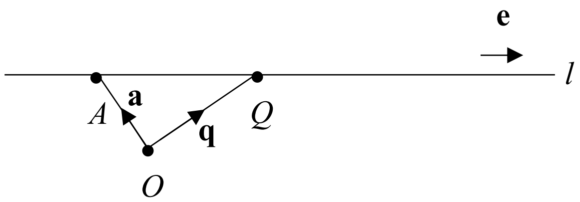
$t = 27.9 \approx 28$ hours (nearest hour)

(iii)



Q2

(i)



<u>**Method 1:**</u>

Equation of l: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{e}, \quad \lambda \in \mathbb{R}$

Since Q lies on l, $\mathbf{q} = \mathbf{a} + \lambda\mathbf{e}, \quad$ for some value of $\lambda \in \mathbb{R}$

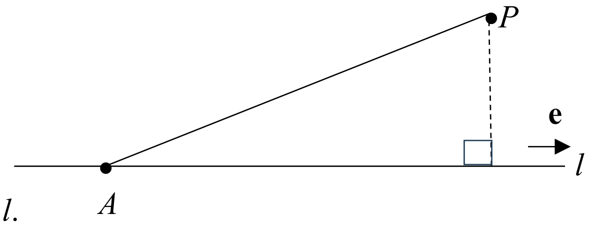
$$\begin{aligned}(\mathbf{q} - \mathbf{a}) \times \mathbf{e} &= (\mathbf{a} + \lambda\mathbf{e} - \mathbf{a}) \times \mathbf{e} \\ &= \lambda\mathbf{e} \times \mathbf{e} \\ &= \mathbf{0} \text{ (Shown)}\end{aligned}$$

<u>**Method 2:**</u>

$\overrightarrow{AQ} = \mathbf{q} - \mathbf{a}$

Since, $\overrightarrow{AQ} // \mathbf{e}$, $(\mathbf{q} - \mathbf{a}) \times \mathbf{e} = \mathbf{0}$. (Shown)

(ii)

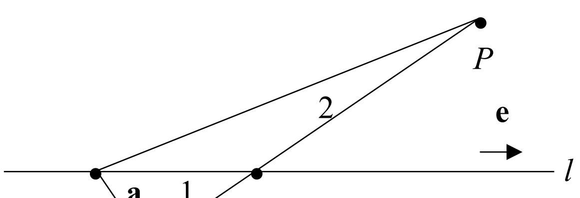


$|(\mathbf{p} - \mathbf{a}) \times \mathbf{e}| = |\overrightarrow{AP} \times \mathbf{e}|$

$|(\mathbf{p} - \mathbf{a}) \times \mathbf{e}|$ represent the perpendicular distance from P to l.

OR area of parallelogram with adjacent sides parallel and equal in magnitude to $\overrightarrow{AP}$ and $\mathbf{e}$

(iii)



Method 1:

Area of triangle $APQ = \frac{1}{2}\left|(\mathbf{p}-\mathbf{a})\times\mathbf{e}\right|\left|\overrightarrow{AQ}\right|$

$$= \frac{1}{2}\left|(3\mathbf{q}-\mathbf{a})\times\mathbf{e}\right|(2)$$
$$= \left|(2\mathbf{q}+\mathbf{q}-\mathbf{a})\times\mathbf{e}\right|$$
$$= \left|(2\mathbf{q})\times\mathbf{e}+(\mathbf{q}-\mathbf{a})\times\mathbf{e}\right|$$
$$= \left|2(\mathbf{q}\times\mathbf{e})+\mathbf{0}\right| \quad \text{(Using (i))}$$
$$= 2\left|\mathbf{q}\times\mathbf{e}\right|$$

Method 2:

Area of triangle $APQ = \frac{1}{2}\left|(\mathbf{p}-\mathbf{a})\times\mathbf{e}\right|\left|\overrightarrow{AQ}\right|$

$$= \frac{1}{2}\left|(3\mathbf{q}-\mathbf{a})\times\mathbf{e}\right|(2)$$
$$= \left|(3\mathbf{q}-\mathbf{q}+\lambda\mathbf{e})\times\mathbf{e}\right| \quad (\text{Since } \mathbf{q}=\mathbf{a}+\lambda\mathbf{e} \Rightarrow \mathbf{a}=\mathbf{q}-\lambda\mathbf{e})$$
$$= \left|(2\mathbf{q})\times\mathbf{e}+\lambda\mathbf{e}\times\mathbf{e}\right|$$
$$= \left|2(\mathbf{q}\times\mathbf{e})+\mathbf{0}\right|$$
$$= 2\left|\mathbf{q}\times\mathbf{e}\right|$$

Method 3:

Area of triangle $APQ = \frac{1}{2}\left|\overrightarrow{AQ}\times\overrightarrow{QP}\right|$ or $\frac{1}{2}\left|\overrightarrow{AQ}\times\overrightarrow{AP}\right|$ or $\frac{1}{2}\left|\overrightarrow{AP}\times\overrightarrow{PQ}\right|$

$$= \frac{1}{2}\left|(\mathbf{q}-\mathbf{a})\times(\mathbf{p}-\mathbf{q})\right|$$
$$= \frac{1}{2}\left|(\lambda\mathbf{e})\times(3\mathbf{q}-\mathbf{q})\right| \quad (\text{Since } \mathbf{q}=\mathbf{a}+\lambda\mathbf{e} \Rightarrow \mathbf{q}-\mathbf{a}=\lambda\mathbf{e})$$
$$= \frac{1}{2}\left|\lambda\right|\left|\mathbf{e}\times(2\mathbf{q})\right|$$
$$= 2\left|\mathbf{q}\times\mathbf{e}\right| \quad (\text{Since } \left|\overrightarrow{AQ}\right|=2,\ \left|\lambda\mathbf{e}\right|=2 \Rightarrow \left|\lambda\right|=2)$$

Note: $\overrightarrow{AQ}=\mathbf{q}-\mathbf{a}=\lambda\mathbf{e}$
Since $\left|\overrightarrow{AQ}\right|=2$,
$\overrightarrow{AQ}=2\mathbf{e}$ or $-2\mathbf{e}$
Hence, $\left|\lambda\right|=2$

(iv)

Method 1:

Area = $2|\mathbf{q}\times\mathbf{e}|=3$

$$2|\mathbf{q}||\mathbf{e}|\sin\theta=3$$
$$2(2)(1)\sin\theta=3$$
$$\sin\theta=\frac{3}{4}$$
$$\theta=48.590°\approx 48.6° \quad \text{(1dp)}$$

Acute angle between l and PQ = Acute angle between $\mathbf{q}$ and $\mathbf{e}$ = $48.6°$

Method 2:

Area = $\frac{1}{2}\left|\overrightarrow{AQ}\right|\left|\overrightarrow{QP}\right|\sin\theta=3$

$$\frac{1}{2}(2)|\mathbf{p}-\mathbf{q}|\sin\theta=3$$
$$|2\mathbf{q}|\sin\theta=3 \qquad \text{Since } \mathbf{p}=3\mathbf{q}$$
$$2|\mathbf{q}|\sin\theta=3$$
$$\sin\theta=\frac{3}{2(2)}$$
$$\theta=48.590°\approx 48.6°$$

Acute angle between l and PQ = Acute angle between $\mathbf{q}$ and $\mathbf{e}$ = $48.6°$

Q3

(i)

$x=\sin\theta+1$ and $y=\sqrt{3}\cos\theta-1$

$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\cos\theta$ and $\frac{\mathrm{d}y}{\mathrm{d}\theta}=-\sqrt{3}\sin\theta$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\sqrt{3}\sin\theta}{\cos\theta}=-\sqrt{3}\tan\theta$ (Shown)

(ii)

At $T\left(\sin t+1,\sqrt{3}\cos t-1\right)$, $\theta=t$

Tangent at T makes an angle of $\frac{3\pi}{4}$ with the positive x-axis: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{3}\tan t=\tan\frac{3\pi}{4}=-1$

$\tan t = \frac{1}{\sqrt{3}}$

$t = \frac{\pi}{6}$

Hence, $T\left(\sin\frac{\pi}{6}+1, \sqrt{3}\cos\frac{\pi}{6}-1\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$

Gradient of normal at $T = 1$

Equation of normal at T: $y - \frac{1}{2} = (1)\left(x - \frac{3}{2}\right)$

$y = x - 1$

(iii)

For tangent to C to be parallel to the x-axis, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{3}\tan\theta = 0 \Rightarrow \theta = 0$

Hence, $y = \sqrt{3}\cos 0 - 1 = \sqrt{3} - 1$

(iv)

At Q,

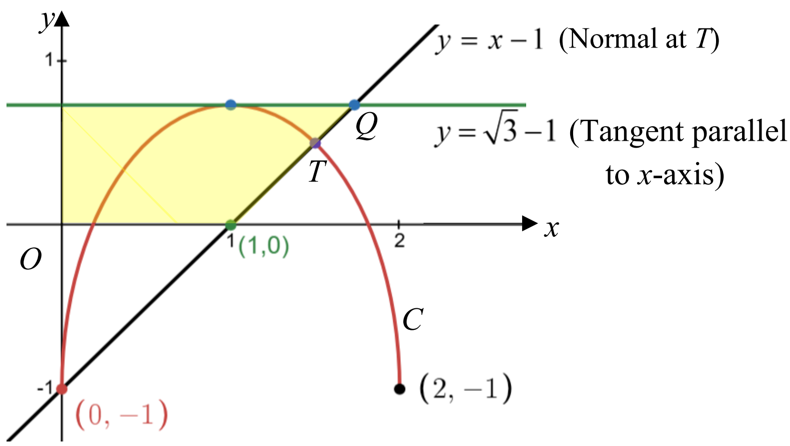
$x - 1 = \sqrt{3} - 1$

$x = \sqrt{3}$

Area of quadrilateral

$= \frac{1}{2}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)$

$= 1$



Q4

(a)

__Method 1__

Since polynomial has only real coefficients, if $3-\mathrm{i}$ is a root, $3+\mathrm{i}$ is another root.

$[w-(3-\mathrm{i})][w-(3+\mathrm{i})]$

$= [(w-3)+\mathrm{i}][(w-3)-\mathrm{i}]$

$= (w-3)^2 - \mathrm{i}^2$

$= w^2 - 6w + 10$

$5w^3 + pw^2 + 68w + q = (w^2 - 6w + 10)(5w + a)$

$= 5w^3 + aw^2 - 30w^2 - 6aw + 50w + 10a$

Comparing coefficients of

$$
\begin{aligned}
w^2 &: \quad a-30=p---(1)\\
w &: \quad -6a+50=68---(2)\\
\text{Constant} &: \quad 10a=q---(3)
\end{aligned}
$$

$$
\begin{aligned}
&\text{From (2)}: \quad -6a+50=68 \Rightarrow a=-3\\
&\text{Subt into (1)}: \ -3-30=p \Rightarrow p=-33\\
&\text{Subt into (3)}: \quad q=-30
\end{aligned}
$$

$$5w^3-33w^2+68w-30=\left(w^2-6w+10\right)\left(5w-3\right)=0$$

The other roots are $3+\mathrm{i}$ and $\dfrac{3}{5}$.

Method 2

$$
\begin{aligned}
&w=3-\mathrm{i}\\
&w^2=\left(3-\mathrm{i}\right)^2=9-6\mathrm{i}+\mathrm{i}^2=8-6\mathrm{i}\\
&w^3=\left(3-\mathrm{i}\right)\left(8-6\mathrm{i}\right)=24-18\mathrm{i}-8\mathrm{i}+6\mathrm{i}^2=18-26\mathrm{i}
\end{aligned}
$$

$$
\begin{aligned}
5w^3+pw^2+68w+q&=0\\
5\left(18-26\mathrm{i}\right)+p\left(8-6\mathrm{i}\right)+68\left(3-\mathrm{i}\right)+q&=0\\
90-130\mathrm{i}+8p-6p\mathrm{i}+204-68\mathrm{i}+q&=0
\end{aligned}
$$

Comparing real and imaginary parts:

$$
\begin{aligned}
&\text{Real}: 90+8p+204+q=0------(1)\\
&\text{Im}: -130-6p-68=0 \Rightarrow p=-33
\end{aligned}
$$

Sub into (1)

$$90+8(-33)+204+q=0 \Rightarrow q=-30$$

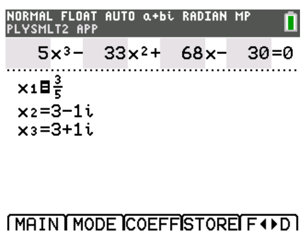
Since polynomial has only real coefficients, if $3-\mathrm{i}$ is a root, $3+\mathrm{i}$ is another root.

$$
\begin{aligned}
&[w-\left(3-\mathrm{i}\right)][w-\left(3+\mathrm{i}\right)]\\
&=[\left(w-3\right)+\mathrm{i}][\left(w-3\right)-\mathrm{i}]\\
&=\left(w-3\right)^2-\mathrm{i}^2\\
&=w^2-6w+10
\end{aligned}
$$

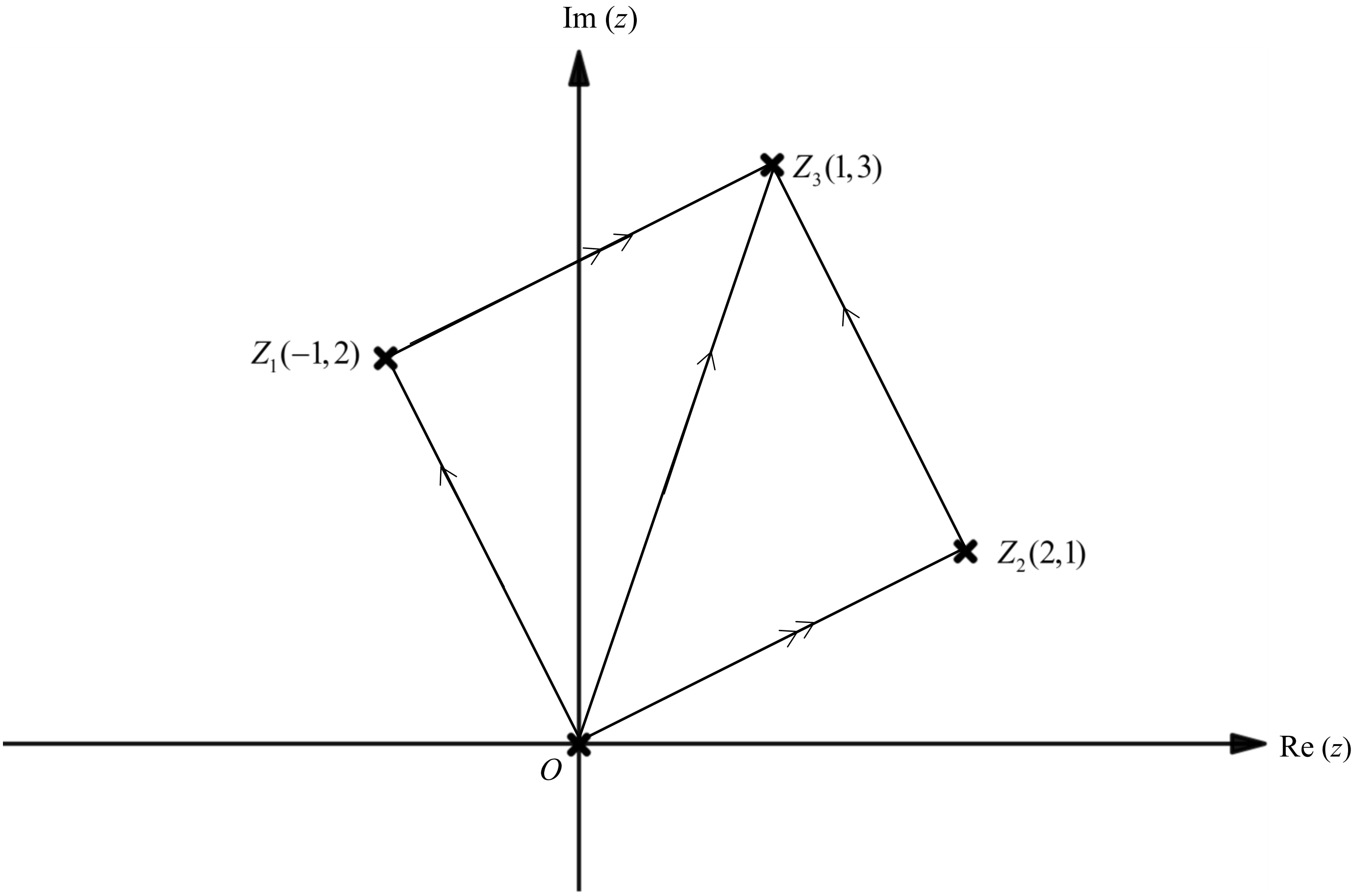
$$5w^3-33w^2+68w-30=\left(w^2-6w+10\right)\left(5w-3\right)=0$$

The other roots are $3+\mathrm{i}$ and $\dfrac{3}{5}$.

Use GC polyroot finder to check the answer.



(b)



(i)

$$\frac{z_1}{z_2}=\frac{-1+2\mathrm{i}}{2+\mathrm{i}}\times\frac{2-\mathrm{i}}{2-\mathrm{i}}=\frac{-2+\mathrm{i}+4\mathrm{i}+2}{5}=\mathrm{i}$$

(ii)

$$\frac{z_1}{z_2}=\mathrm{i}\Rightarrow z_1=\mathrm{i}z_2$$

Rotate Z_2 by $\frac{\pi}{2}$ radians anti-clockwise about O to get Z_1

(1) $\therefore Z_1OZ_2$ is a right angle.

(2) $OZ_1=OZ_2$ (since $z_1=\mathrm{i}z_2$ or since $|z_1|=|-1+2\mathrm{i}|=\sqrt{5}$ and $|z_2|=|2+\mathrm{i}|=\sqrt{5}$)

(3) Also, since $\overrightarrow{OZ_3}=\overrightarrow{OZ_1}+\overrightarrow{OZ_2}$, $OZ_1Z_3Z_2$ is a parallelogram.

From (1) + (2) + (3), we can deduce that $OZ_2Z_3Z_1$ is a square.

Q5

(i)

Number of ways to draw 4 orbs = ${}^{12}C_4 = 495$

(ii)

Number of ways to draw at least 2 colours = $495 - {}^{5}C_4 - {}^{4}C_4 = 489$

(iii)

Case 1: 2R, 1B, 1G

Number of ways = ${}^{5}C_2 \times {}^{4}C_1 \times {}^{3}C_1 = 120$

Case 2: 1R, 2B, 1G

Number of ways = ${}^{5}C_1 \times {}^{4}C_2 \times {}^{3}C_1 = 90$

Case 3: 1R, 1B, 2G

Number of ways = ${}^{5}C_1 \times {}^{4}C_1 \times {}^{3}C_2 = 60$

Total number of ways = 120 + 90 + 60 = 270

Q6

(i)

The team should conduct a 1-tail test as they are verifying the claim that users are spending more than 15 minutes.

(ii)

Central Limit Theorem states that sample means will follow a normal distribution approximately when the sample size is more than 30. Since the sample size is 80, the team is able to carry out a hypothesis test without knowing anything about the distribution of the times spend by the users.

(iii)

X: time spent per visit
μ : population mean time spent per visit

$H_0 : \mu = 15$
$H_1 : \mu > 15$

Under H_0, since $n = 80$ is large, by Central Limit Theorem,
$\overline{X} \sim N\left(15, \dfrac{\sigma^2}{80}\right)$ approximately

Test statistic, $z = \dfrac{\overline{x} - \mu}{\sqrt{\dfrac{\sigma^2}{n}}} = \dfrac{16 - 15}{\sqrt{\dfrac{\sigma^2}{80}}} = \dfrac{1}{\sqrt{\dfrac{\sigma^2}{80}}}$

For 5% level of significance, reject H_0 if $z > 1.6449$

In order for platform's claim to be valid, H_0 must be rejected, hence,

$$\frac{1}{\sqrt{\frac{\sigma^2}{80}}} > 1.6449$$

$$\sqrt{\frac{\sigma^2}{80}} < 0.6079396$$

$$0 < \sigma < 5.44$$

(iv)

There is a probability of 0.05 of concluding that the population mean time spent per visit is more than 15 minutes when the population mean time spent is 15 minutes.

Q7

(i)

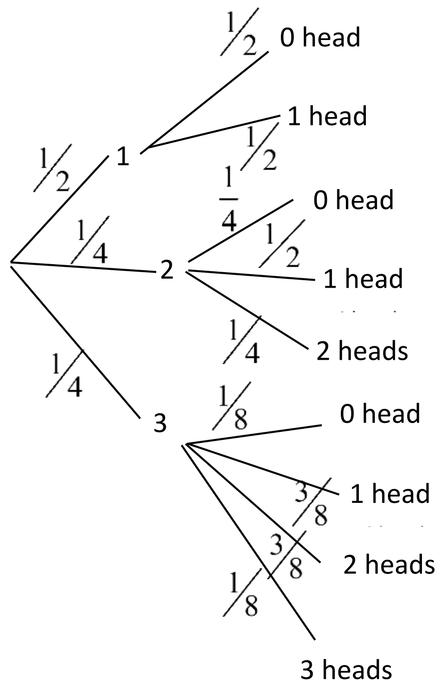
$$a = \mathrm{P}(X=0) = \left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{4}\times\left(\frac{1}{2}\right)^2\right)+\left(\frac{1}{4}\times\left(\frac{1}{2}\right)^3\right)=\frac{11}{32}$$

$$b = \mathrm{P}(X=2) = \left(\frac{1}{4}\times\left(\frac{1}{2}\right)^2\right)+\left(\frac{1}{4}\times {}^3C_2\times\left(\frac{1}{2}\right)^3\right)=\frac{5}{32}$$

Or

$$b = \mathrm{P}(X=2) = 1-\frac{11}{32}-\frac{15}{32}-\frac{1}{32}=\frac{5}{32}$$

$$a=\frac{11}{32},\ b=\frac{5}{32}$$



(ii)

Probability distribution of X:

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{11}{32}$	$\frac{15}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$$\mathrm{E}(X)=\sum x\mathrm{P}(X=x)=(0)\left(\frac{11}{32}\right)+(1)\left(\frac{15}{32}\right)+(2)\left(\frac{5}{32}\right)+(3)\left(\frac{1}{32}\right)=\frac{7}{8}$$

$$\mathrm{E}(X^2)=\sum x^2\mathrm{P}(X=x)=(0)^2\left(\frac{11}{32}\right)+(1)^2\left(\frac{15}{32}\right)+(2)^2\left(\frac{5}{32}\right)+(3)^2\left(\frac{1}{32}\right)=\frac{11}{8}$$

$$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2=\frac{11}{8}-\left(\frac{7}{8}\right)^2=\frac{39}{64}\text{ (Shown)}$$

(iii)

Let X = number of heads obtained in one game

$\bar{X}$ = mean number of heads obtained per game

Since n =50 is large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\frac{7}{8}, \frac{39/64}{50}\right) = \mathrm{N}\left(\frac{7}{8}, \frac{39}{3200}\right)$ approximately

$\mathrm{P}\left(\bar{X} > 1\right) = 0.12876 \approx 0.129$

Q8

(i)

(1)The condition of a portable speaker is independent of the condition of any other portable speakers.

Or

Whether a portable speaker is faulty is independent of the condition of any other portable speakers.

(2) The probability that any portable speaker is faulty is constant.

(ii)

Let X = number of faulty portable speakers out of 24

$X \sim \mathrm{B}(24, 0.02)$

$\mathrm{P}(X \le 1) = 0.917387 \approx 0.917$

(iii)

Method 1:

Let Y = number of substandard cartons out of 50

$\mathrm{P}(X > 1) = 1 - 0.917387 = 0.082613$

$Y \sim \mathrm{B}(50, 0.082613)$

$\mathrm{P}(Y = 0) = 0.013416 \approx 0.0134$

Method 2:

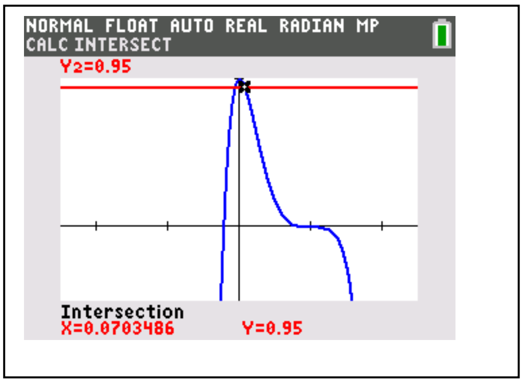
Required Probability $= \left[\mathrm{P}(X \le 1)\right]^{50} = \left[0.917387\right]^{50} = 0.0134$

(iv)

Let F = the number of faulty flash drives out of 3

$F \sim \mathrm{B}(3, p)$

P(batch is accepted)



$=\mathrm{P}(F=0)+\mathrm{P}(F=1)\times\mathrm{P}(F=0)$

$=(1-p)^3+{}^3C_1p^1(1-p)^2\times(1-p)^3$

$=(1-p)^3\left[1+3p(1-p)^2\right]$

At least 95% of the batches are accepted

$(1-p)^3\left[1+3p(1-p)^2\right]\geq 0.95$

From the GC, $0<p\leq 0.0703$

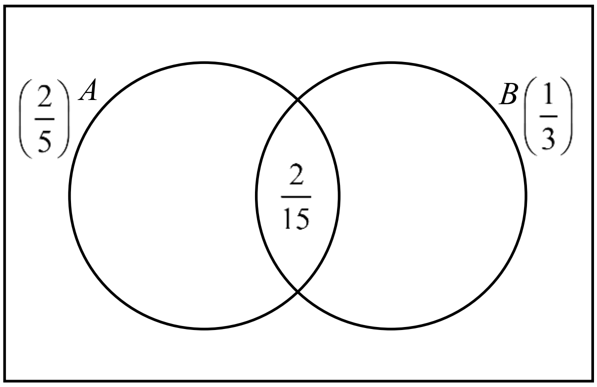
Q9

(i)

Given $\mathrm{P}(A|B)=\frac{2}{5}$,

$$\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{2}{5}$$

$$\mathrm{P}(A\cap B)=\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}$$



Also, $\mathrm{P}(A\cup B)=\frac{3}{5}$

$$\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=\frac{3}{5}$$

$$\mathrm{P}(A)+\frac{1}{3}-\frac{2}{15}=\frac{3}{5}$$

$$\mathrm{P}(A)=\frac{2}{5}$$

Since $\mathrm{P}(A)=\mathrm{P}(A|B)=\frac{2}{5}$, events A and B <u>are</u> independent.

Or

Since $\mathrm{P}(A)\cdot\mathrm{P}(B)=\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}=\mathrm{P}(A\cap B)$, events A and B <u>are</u> independent.

(ii)

$\mathrm{P}(A'\cap B)=\mathrm{P}(A\cup B)-\mathrm{P}(A)$ **or** $\mathrm{P}(A'\cap B)=\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$=\frac{3}{5}-\frac{2}{5}=\frac{1}{5}$ $\qquad$ $=\frac{1}{3}-\frac{2}{15}=\frac{1}{5}$

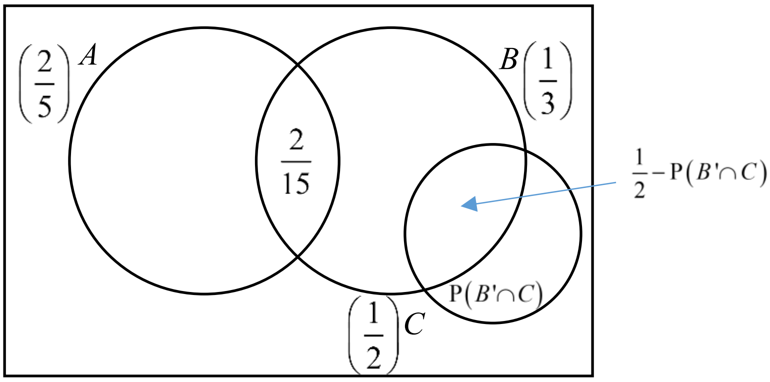
Or since A and B are independent, A' and B are also independent.

$$\mathrm{P}(A'\cap B)=\mathrm{P}(A')\times\mathrm{P}(B)$$
$$=\frac{3}{5}\times\frac{1}{3}=\frac{1}{5}$$

(iii)

<u>Method 1:</u>

Since A and C are mutually exclusive, we have



From the Venn diagram,

$$\mathrm{P}(B\cap C)\le\mathrm{P}(A'\cap B)$$
$$\frac{1}{2}-\mathrm{P}(B'\cap C)\le\frac{1}{5}$$
$$\mathrm{P}(B'\cap C)\ge\frac{1}{2}-\frac{1}{5}=\frac{3}{10}$$

Also, $\mathrm{P}(A\cup B\cup C)\le 1$

$$\mathrm{P}(A\cup B)+\mathrm{P}(B'\cap C)\le 1$$
$$\frac{3}{5}+\mathrm{P}(B'\cap C)\le 1$$
$$\mathrm{P}(B'\cap C)\le\frac{2}{5}$$

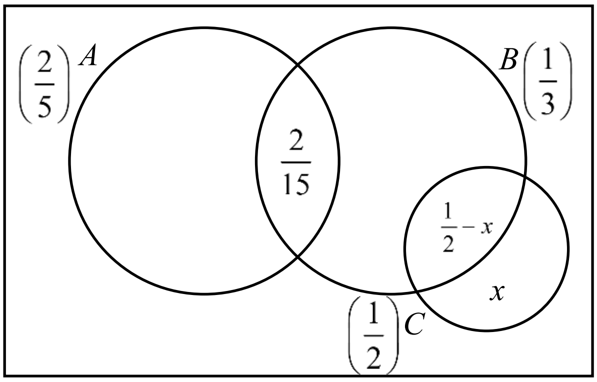
Hence, the least possible value of $\mathrm{P}(B'\cap C)=\frac{3}{10}$

And the greatest possible value of $\mathrm{P}(B'\cap C)=\frac{2}{5}$.

<u>Method 2:</u>

Let $\mathrm{P}(B'\cap C)=x$

$$\mathrm{P}(B\cap C)=\frac{1}{2}-x$$
$$\mathrm{P}(A'\cap B\cap C')=\mathrm{P}(A'\cap B)-\mathrm{P}(B\cap C)$$
$$=\frac{1}{5}-\left(\frac{1}{2}-x\right)$$
$$=x-\frac{3}{10}$$
$$\mathrm{P}(A'\cap B'\cap C')=1-\mathrm{P}(A\cup B)-\mathrm{P}(B'\cap C)$$
$$=1-\frac{3}{5}-x$$
$$=\frac{2}{5}-x$$



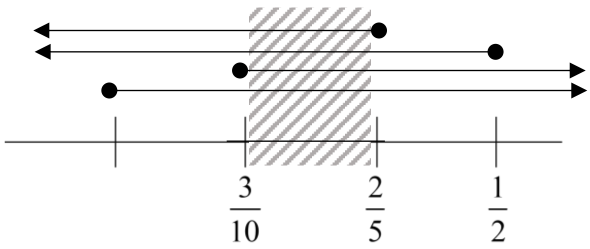
Since they are all probability, we have

$$x \geq 0$$

$$\frac{1}{2} - x \geq 0 \quad \Rightarrow \quad x \leq \frac{1}{2}$$

$$x - \frac{3}{10} \geq 0 \quad \Rightarrow \quad x \geq \frac{3}{10}$$

$$\frac{2}{5} - x \geq 0 \quad \Rightarrow \quad x \leq \frac{2}{5}$$



Solving the inequalities, $\frac{3}{10} \leq x \leq \frac{2}{5}$.

Hence, the least possible value of $\mathrm{P}\left(B' \cap C\right) = \frac{3}{10}$

And the greatest possible value of $\mathrm{P}\left(B' \cap C\right) = \frac{2}{5}$.

Q10

(i)

Let A = length of a ball of 2-ply yarn from Machine A

$$A \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

$$\mathrm{P}\left(A < 340\right) = 0.8 \qquad \mathrm{P}\left(A < 280\right) = 0.1$$

$$\mathrm{P}\left(Z < \frac{340 - \mu}{\sigma}\right) = 0.8 \qquad \mathrm{P}\left(Z < \frac{280 - \mu}{\sigma}\right) = 0.1$$

$$\frac{340 - \mu}{\sigma} = 0.84162 \qquad \& \qquad \frac{280 - \mu}{\sigma} = -1.28155$$

$$340 - \mu = 0.84162\sigma \qquad 280 - \mu = -1.28155\sigma$$

$$\mu + 0.84162\sigma = 340 ----(1) \qquad \mu - 1.28155\sigma = 280 ----(2)$$

Solve (1) and (2)

$$\mu = 316.216 \approx 316$$
$$\sigma = 28.259 \approx 28.3 \quad \text{(AG)}$$

(ii)

Let B = length of a ball of 3 ply yarn from Machine B

$$B \sim \mathrm{N}\left(220, 6^2\right)$$

$$\left[\mathrm{P}\left(B > 230\right)\right]^2 \times \left[\mathrm{P}\left(B < 215\right)\right]^3 \times \frac{5!}{3!2!} = 0.000189169$$
$$\approx 0.000189$$

(iii)

$A \sim \mathrm{N}\left(300, 20^2\right), \quad B \sim \mathrm{N}\left(220, 6^2\right)$

$T = A_1 + A_2 + B_1 + B_2 + B_3 \sim \mathrm{N}\left(2\times 300 + 3\times 220, 2\times 20^2 + 3\times 6^2\right) = \mathrm{N}\left(1260, 908\right)$

$\mathrm{P}\left(T > k\right) = 0.385$
$k = 1268.81 \quad \text{(2dp)}$

(iv)

$A \sim \mathrm{N}\left(300, 20^2\right), \quad B \sim \mathrm{N}\left(220, 6^2\right)$

$X = 0.91(A_1 + A_2 + A_3) \sim \mathrm{N}\left(0.91\times 3\times 300,\ 0.91^2 \times 3\times 20^2\right) = \mathrm{N}\left(819, 993.72\right)$
$Y = 0.95(B_1 + \cdots + B_4) \sim \mathrm{N}\left(0.95\times 4\times 220, 0.95^2 \times 4\times 6^2\right) = \mathrm{N}\left(836, 129.96\right)$

$X - Y \sim \mathrm{N}\left(819 - 836, \quad 993.72 + 129.96\right) = \mathrm{N}(-17, 1123.68)$

$$
\begin{aligned}
\mathrm{P}\left(\left|X - Y\right| < 50\right) &= \mathrm{P}\left(-50 < X - Y < 50\right) \\
&= 0.814733 \\
&\approx 0.815
\end{aligned}
$$

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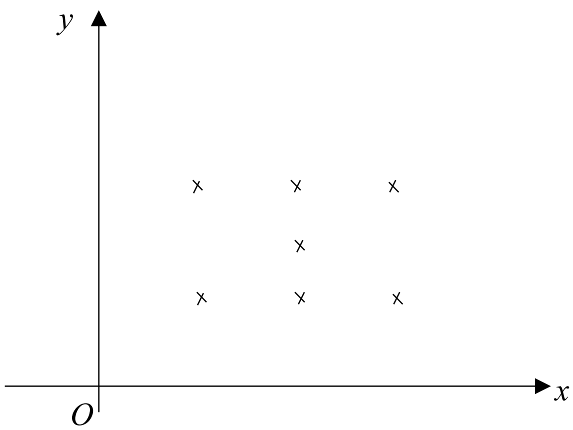
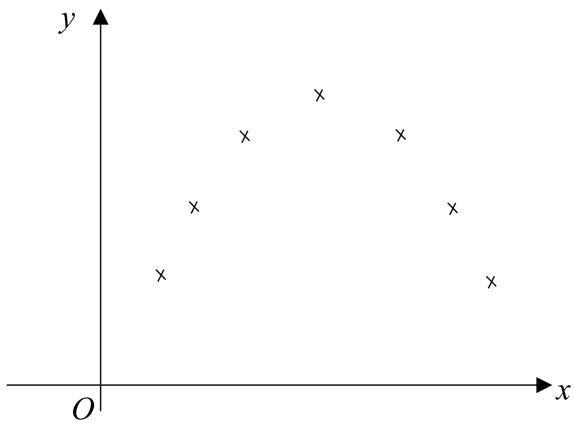
(a)(i)

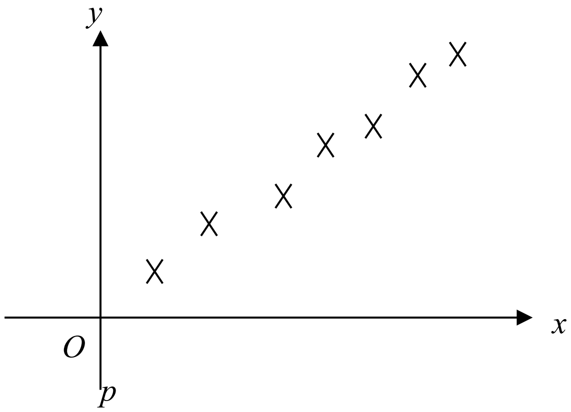
Possible scatter diagrams:



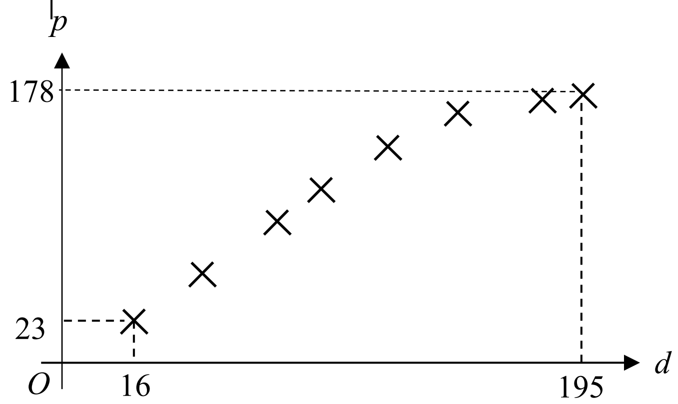


(a)(ii)

One possible scatter diagram:



(b)(i)



(b)(ii)

The values of the residuals may be positive or negative. To avoid the cancellation effect in adding them, the squares of the residuals, which are non-negative, should be used.

(b)(iii)

The linear relationship is not appropriate as the scatter diagram indicates that as *d* increases, *p* increases at a decreasing rate.

(b)(iv)

$$p = -184.5953136 + 68.95820682 \ln d$$
$$\approx -185 + 69.0 \ln d \text{ (to 3 sf)}$$
$$p = -184.5953136 + 68.95820682 \ln(210) \approx 184.13 \text{ (to 2 dp)}$$

(b)(v)

Since the range of values of *d* is from 16 to 195, $d = 210$ is out of the given data range. To estimate the value of *p* when *d* = 210 is an extrapolation, which may not necessary be reliable as the linear relationship between ln *d* and *p* may not hold at the extrapolated values.

(b)(vi)

The value of *t* will remain the same while the value of *s* will increase by 10.